CS 61B
Spring 2026

Shortest Paths and MSTs

Exam-Level Discussion 10: March 30, 2026

1 Sticky Flights

Your airline company has been contracted to fly a large shipment of honey from Honeysville to the 61Bees in Goldenhive City. However, the airplane doesn't have enough fuel capacity to fly directly to Goldenhive City so it will stop at at least one of n airports along the way to refuel. Refueling takes an hour, and if the airport is one of $k < n$ airports, your airplane will be grounded for six hours due to curfews (refueling is included in the six hours). The 61Bees want their honey as soon as possible so please design an algorithm to find the route that will allow your airplane to reach Goldenhive City in the least amount of hours.

Hint: Think of the n airports as a graph, where the paths between them are edges of weight equivalent to the number of hours it takes to fly from airport A to airport B. You may assume that the amount of time it takes to fly from A to B is equal to the amount of time it takes to fly from B to A.

2 Multiple MSTs

Recall a graph can have multiple MSTs if there are multiple spanning trees of minimum weight.

(a) For each subpart below, select the correct option and justify your answer. If you select "never" or "always," provide a short explanation. If you select "sometimes," provide two graphs that fulfill the given properties.

1. If **some** of the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

2. If **all** of the edge weights are **identical**, there will

   ◯ never be multiple MSTs in G.

   ◯ sometimes be multiple MSTs in G.

   ◯ always be multiple MSTs in G.

   Justification:

(b) Suppose we have a connected, undirected graph (G) with (N) vertices and (N) edges, where all the **edge weights are identical**. Find the maximum and minimum number of MSTs in (G) and explain your reasoning.

Minimum:

Maximum:

Justification:

(c) It is possible that Prim's and Kruskal's find different MSTs on the same graph G (as an added exercise, construct a graph where this is the case!).

Given any graph G with integer edge weights, modify the edge weights of G to *ensure* that

(1) Prim's and Kruskal's will output the same results, and

(2) the output edges still form a MST correctly in the original graph.

You may not modify Prim's or Kruskal's, and you may not add or remove any nodes/edges.

Hint: Look at subpart 1 of part (a).

3 Sticky Railroads

Two cities, Chicago and Berkeley, are located in the United States. The railroad system connecting them can be modeled as a **weighted directed graph**, with V vertices, E edges, and weights representing the length of the railway. Ethan wishes to take a railway from Chicago to Berkeley, and needs to determine the shortest railway distance between them.

Define the set C to be all cities in Chicago, and B to be all cities in Berkeley. There can be cities that belong to neither region along the way. The shortest distance between the two cities is the shortest distance between any city c_C in Chicago and c_B in Berkeley.

Describe an algorithm that computes the minimum railway distance from Chicago to Berkeley, in $O((V + E) \log V)$ time. You are able to utilize all graph algorithms you learned in class.

Hint: Consider modifying the graph so that running a graph algorithm yields an equivalent answer to solving the original problem.